Testing the Difference Between Means - Large Samples

OBJECTIVES

- How to decide whether two samples are independent or dependent
- An introduction to two-sample hypothesis testing for the difference between two population parameters
- How to perform a two-sample *z*-test for the difference between two means μ_1 and μ_2 using large independent samples

VOCABULARY

- Dependent samples
- Independent samples
- Two-sample z-test

The standardized test statistic is

$$z = \frac{(\overline{x}_1 - \overline{x}_2) - (\mu_1 - \mu_2)}{\sigma_{\overline{x}_1 - \overline{x}_2}}$$

$$\sigma_{\overline{x}_1 - \overline{x}_2} \approx \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

null and alternative hypotheses

$$\begin{cases} H_0\colon \mu_1 = \mu_2 \\ H_a\colon \mu_1 \neq \mu_2 \end{cases}, \quad \begin{cases} H_0\colon \mu_1 \leq \mu_2 \\ H_a\colon \mu_1 > \mu_2 \end{cases}, \quad \text{and} \quad \begin{cases} H_0\colon \mu_1 \geq \mu_2 \\ H_a\colon \mu_1 < \mu_2 \end{cases}.$$

When you compare the means of two different populations, the method you use to sample as well as the sample sizes will determine the type of test you will use. For large independent samples, ($n \geq 30$) the *z*-test will be used. For dependent small samples, ($n < 30$) the *t*-test will be used.

INSIGHT

Dependent samples often involve identical twins, before and after results for the same person or object, or results of individuals matched for specific characteristics.

AN OVERVIEW OF TWO-SAMPLE HYPOTHESIS TESTING

It is important to remember that when you perform a two-sample hypothesis test using independent samples, you are testing a claim concerning the difference between the parameters in two populations, not the values of the parameters themselves.

DEFINITION

For a two-sample hypothesis test with independent samples,

1. the **null hypothesis** H_0 is a statistical hypothesis that usually states there is no difference between the parameters of two populations. The null hypothesis always contains the symbol $\leq$, $=$, or $\geq$.
2. the **alternative hypothesis** H_a is a statistical hypothesis that is true when H_0 is false. The alternative hypothesis contains the symbol $>$, $\neq$, or $<$.

STUDY TIP

You can also write the null and alternative hypotheses as follows.

$$\begin{cases} H_0: \mu_1 - \mu_2 = 0 \\ H_a: \mu_1 - \mu_2 \neq 0 \end{cases}$$

$$\begin{cases} H_0: \mu_1 - \mu_2 \leq 0 \\ H_a: \mu_1 - \mu_2 > 0 \end{cases}$$

$$\begin{cases} H_0: \mu_1 - \mu_2 \geq 0 \\ H_a: \mu_1 - \mu_2 < 0 \end{cases}$$

<u>null and alternative hypotheses</u>

$$\begin{cases} H_0: \mu_1 = \mu_2 \\ H_a: \mu_1 \neq \mu_2 \end{cases}, \quad \begin{cases} H_0: \mu_1 \leq \mu_2 \\ H_a: \mu_1 > \mu_2 \end{cases}, \quad \text{and} \quad \begin{cases} H_0: \mu_1 \geq \mu_2 \\ H_a: \mu_1 < \mu_2 \end{cases}.$$

AN OVERVIEW OF TWO-SAMPLE HYPOTHESIS TESTING

Three conditions are necessary to perform such a test.

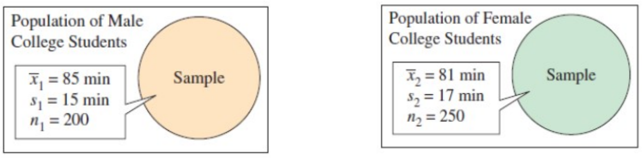
1. The samples must be randomly selected.
2. The samples must be independent.
3. Each sample size must be at least 30 or, if not, each population must have a normal distribution with a known standard deviation.

If these requirements are met, then the **sampling distribution for $\overline{x}_1 - \overline{x}_2$, the difference of the sample means,** is a normal distribution with mean and standard error as follows.

In Words	In Symbols
The mean of the difference of the sample means is the assumed difference between the two population means. When no difference is assumed, the mean is 0.	Mean $= \mu_{\overline{x}_1 - \overline{x}_2} = \mu_{\overline{x}_1} - \mu_{\overline{x}_2} = \mu_1 - \mu_2$
The variance of the sampling distribution is the sum of the variances of the individual sampling distributions for $\overline{x}_1$ and $\overline{x}_2$. The standard error is the square root of the sum of the variances.	Standard error $= \sigma_{\overline{x}_1 - \overline{x}_2} = \sqrt{\sigma_{\overline{x}_1}^2 + \sigma_{\overline{x}_2}^2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

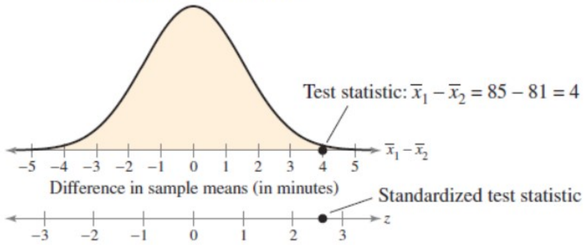
AN OVERVIEW OF TWO-SAMPLE HYPOTHESIS TESTING

The average time college students spend online



The graph below shows the sampling distribution of $\overline{x}_1 - \overline{x}_2$ for many similar samples taken from two populations for which $\mu_1 - \mu_2 = 0$. From the graph, you can see that it is quite unlikely to obtain sample means that differ by 4 minutes if the actual difference is 0. The difference of the sample means would be more than 2.5 standard errors from the hypothesized difference of 0! So, you can conclude that there is a significant difference in the amounts of time male college students and female college students spend online each day.

Sampling Distribution



INSIGHT

The members in the two samples are not matched or paired, so the samples are independent.

null and alternative hypotheses

$$\begin{cases} H_0: \mu_1 = \mu_2 \\ H_a: \mu_1 \neq \mu_2, \end{cases}$$

EXAMPLE 1 Independent and Dependent Samples

Classify each pair of samples as independent or dependent and justify your answer.

1. Sample 1: Weights of 65 college students before their freshman year begins
   Sample 2: Weights of the same 65 college students after their freshman year
2. Sample 1: Scores for 38 adult males on a psychological screening test for attention-deficit hyperactivity disorder
   Sample 2: Scores for 50 adult females on a psychological screening test for attention-deficit hyperactivity disorder

Solution

1. These samples are dependent. Because the weights of the same students are taken, the samples are related. The samples can be paired with respect to each student.
2. These samples are independent. It is not possible to form a pairing between the members of samples, the sample sizes are different, and the data represent scores for different individuals.

▸ Try It Yourself 1

Classify each pair of samples as independent or dependent.

1. Sample 1: Systolic blood pressures of 30 adult females
   Sample 2: Systolic blood pressures of 30 adult males
2. Sample 1: Midterm exam scores of 14 chemistry students
   Sample 2: Final exam scores of the same 14 chemistry students

a. Determine whether the samples are *independent* or *dependent*.
b. *Explain* your reasoning.

TWO-SAMPLE z-TEST FOR THE DIFFERENCE BETWEEN MEANS

GUIDELINES

Using a Two-Sample z-Test for the Difference Between Means (Large Independent Samples)

IN WORDS	IN SYMBOLS
1. State the claim mathematically and verbally. Identify the null and alternative hypotheses.	State H_0 and H_a.
2. Specify the level of significance.	Identify α.
3. Determine the critical value(s).	Use Table 4 in Appendix B.
4. Determine the rejection region(s).	
5. Find the standardized test statistic and sketch the sampling distribution.	$z = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sigma_{\bar{x}_1 - \bar{x}_2}}$
6. Make a decision to reject or fail to reject the null hypothesis.	If z is in the rejection region, reject H_0. Otherwise, fail to reject H_0.
7. Interpret the decision in the context of the original claim.	

A hypothesis test for the difference between means can also be performed using P-values. Use the guidelines above, skipping Steps 3 and 4. After finding the standardized test statistic, use the Standard Normal Table to calculate the P-value. Then make a decision to reject or fail to reject the null hypothesis. If P is less than or equal to α, reject H_0. Otherwise, fail to reject H_0.

Testing the Difference Between Means - Large Samples

EXAMPLE 2 A Two-Sample z-Test for the Difference Between Means

A credit card watchdog group claims that there is a difference in the mean credit card debts of households in New York and Texas. The results of a random survey of 250 households from each state are shown at the left. The two samples are independent. Do the results support the group's claim? Use $\alpha = 0.05$. *(Adapted from PlasticRewards.com)*

Solution

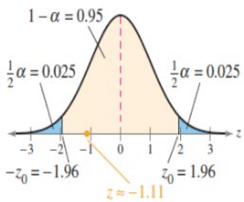
The claim is "there is a difference in the mean credit card debts of households in New York and Texas." So, the null and alternative hypotheses are

$H_0: \mu_1 = \mu_2$ and $H_a: \mu_1 \neq \mu_2$. (Claim)

Because the test is a two-tailed test and the level of significance is $\alpha = 0.05$, the critical values are $-z_0 = -1.96$ and $z_0 = 1.96$. The rejection regions are $z < -1.96$ and $z > 1.96$. Because both samples are large, s_1 and s_2 can be used in place of σ_1 and σ_2 to calculate the standard error.

$$\sigma_{\bar{x}_1-\bar{x}_2} \approx \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$= \sqrt{\frac{1045.70^2}{250} + \frac{1361.95^2}{250}} \approx 108.5983$$



The standardized test statistic is

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sigma_{\bar{x}_1-\bar{x}_2}}$$ Use the z-test.

$$\approx \frac{(4446.25 - 4567.24) - 0}{108.5983}$$ Assume $\mu_1 = \mu_2$, so $\mu_1 - \mu_2 = 0$.

$$\approx -1.11.$$

The graph at the left shows the location of the rejection regions and the standardized test statistic z. Because z is not in the rejection region, you should fail to reject the null hypothesis.

Interpretation There is not enough evidence at the 5% level of significance to support the group's claim that there is a difference in the mean credit card debts of households in New York and Texas.

Try It Yourself 2

A survey indicates that the mean annual wages for forensic science technicians working for local and state governments are \$53,300 and \$51,910, respectively. The survey includes a randomly selected sample of size 100 from each government branch. The sample standard deviations are \$6200 (local) and \$5575 (state). The two samples are independent. At $\alpha = 0.10$, is there enough evidence to conclude that there is a difference in the mean annual wages? *(Adapted from U.S. Bureau of Labor Statistics)*

a. Identify the *claim* and state H_0 and H_a.
b. Identify the *level of significance* α.
c. Find the *critical values* and identify the *rejection regions*.
d. Use the z-test to find the *standardized test statistic* z. *Sketch* a graph.
e. *Decide* whether to reject the null hypothesis.
f. *Interpret* the decision in the context of the original claim.

EXAMPLE 3

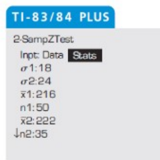
A travel agency claims that the average daily cost of meals and lodging for vacationing in Texas is less than the same average cost for vacationing in Virginia. The table at the left shows the results of a random survey of vacationers in each state. The two samples are independent. At $\alpha = 0.01$, is there enough evidence to support the claim? [H_0: $\mu_1 \geq \mu_2$ and H_a: $\mu_1 < \mu_2$ (claim)] *(Adapted from American Automobile Association)*

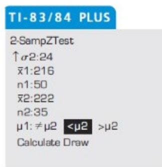
Sample Statistics for Daily Cost of Meals and Lodging for Two Adults

Texas	Virgina
$\bar{x}_1 = \$216$	$\bar{x}_2 = \$222$
$s_1 = \$18$	$s_2 = \$24$
$n_1 = 50$	$n_2 = 35$

Solution

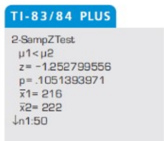
The top two displays show how to set up the hypothesis test using a TI-83/84 Plus. The remaining displays show the possible results, depending on whether you select *Calculate* or *Draw*.

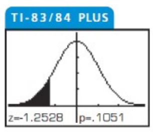




Because the test is a left-tailed test and $\alpha = 0.01$, the rejection region is $z < -2.33$. The standardized test statistic $z \approx -1.25$ is not in the rejection region, so you should fail to reject the null hypothesis.

Interpretation There is not enough evidence at the 1% level of significance to support the travel agency's claim.





▸ Try It Yourself 3

A travel agency claims that the average daily cost of meals and lodging for vacationing in Alaska is greater than the same average cost for vacationing in Colorado. The table at the left shows the results of a random survey of vacationers in each state. The two samples are independent. At $\alpha = 0.05$, is there enough evidence to support the claim? *(Adapted from American Automobile Association)*

Sample Statistics for Daily Cost of Meals and Lodging for Two Adults

Alaska	Colorado
$\bar{x}_1 = \$274$	$\bar{x}_2 = \$271$
$s_1 = \$22$	$s_2 = \$18$
$n_1 = 150$	$n_2 = 200$

a. Use a TI-83/84 Plus to find the *test statistic* or the *P-value*.
b. *Decide* whether to reject the null hypothesis.
c. *Interpret* the decision in the context of the original claim.

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VOCABULARY

- Dependent samples
- Independent samples
- Two-sample z-test

CLASSWORK

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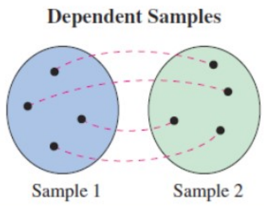

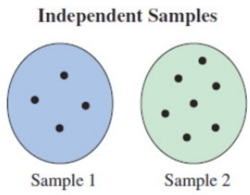

INSIGHT

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null and alternative hypotheses

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